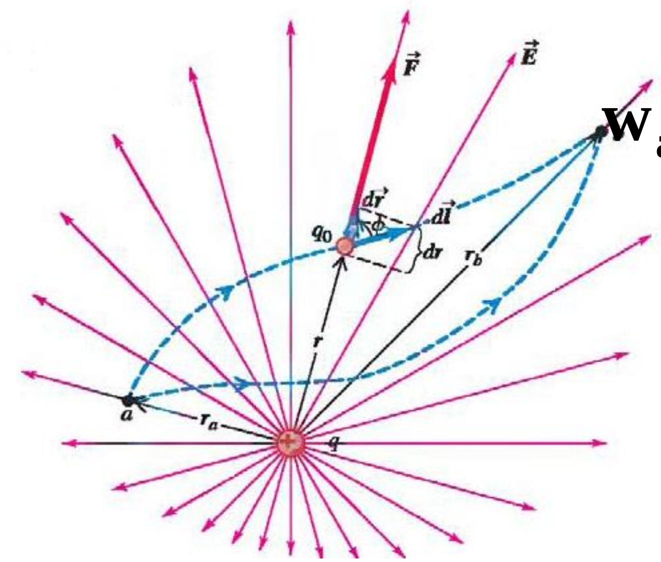
Fisica II: Potencial

Profesora : Dra. Elsa Hogert

- **Bibliografía consultada: Sears- Zemasnky -Tomo II**
  Serway- Jewett – Tomo II

POTENCIAL ELECTROSTÁTICO



$$W_{a\to b} = \int_{r_a}^{r_b} \vec{F}.d\vec{l} = \int_{r_a}^{r_b} q_0\,\vec{E}.d\vec{l} = q_0\int_{r_a}^{r_b} \vec{E}.d\vec{l} = -\Delta U$$

Potencial Electrostático

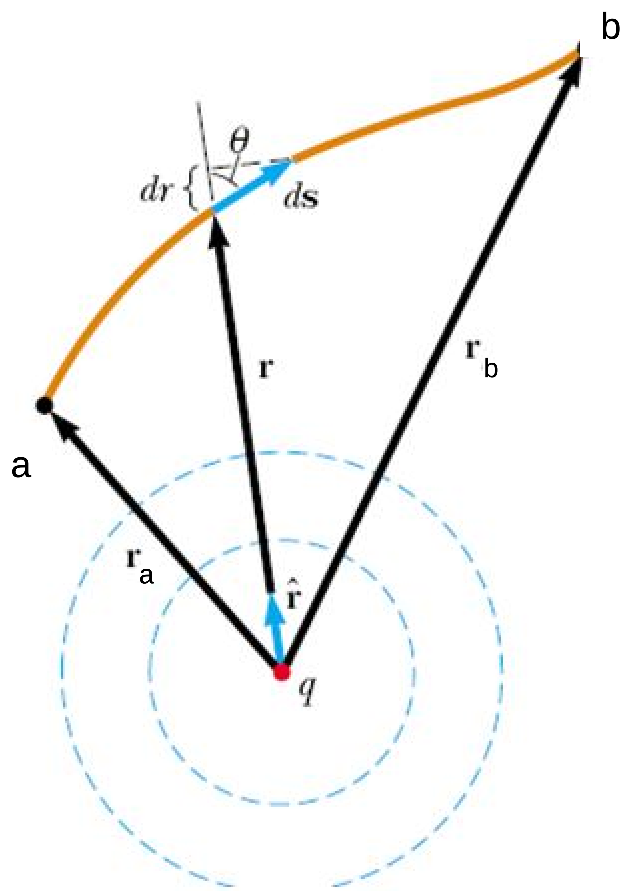
$$\frac{\Delta U}{q_0} = \Delta V = V_b - V_a = -\int_{r_a}^{r_b} \vec{E}.d\vec{l}$$

$$[V] = \frac{J}{C} = \frac{N.m}{C} = V = VOLT$$

1. Energía potencial por unidad de carga.
2. Menos el Trabajo realizado por **E** para desplazar una carga de pueba desde **a** hasta **b**.
3. Trabajo por unidad de carga realizado por una fuerza externa.

$$\oint \vec{E}.d\vec{l} = 0$$

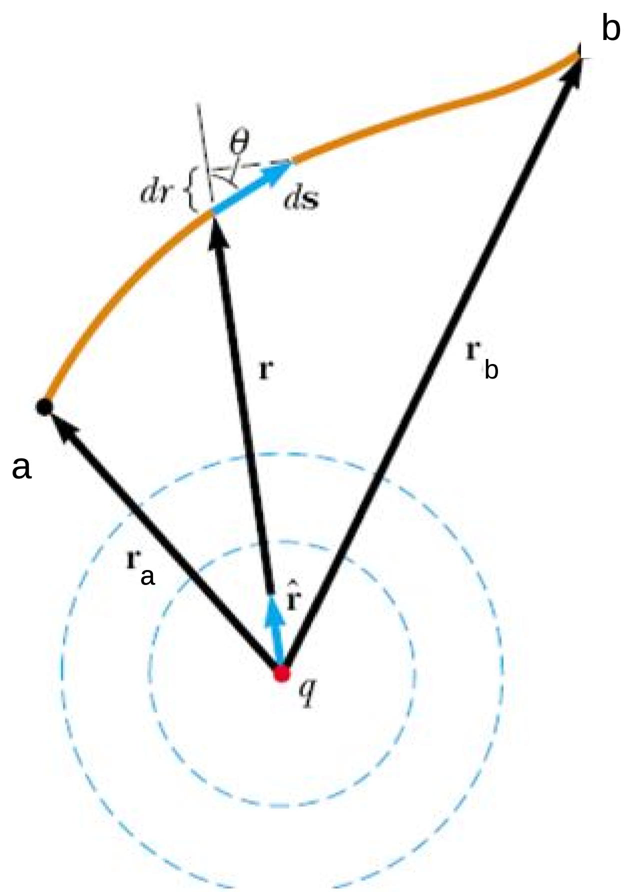
$$\Delta V = V_b - V_a = -\int_{r_a}^{r_b} \vec{E}.d\vec{l}$$



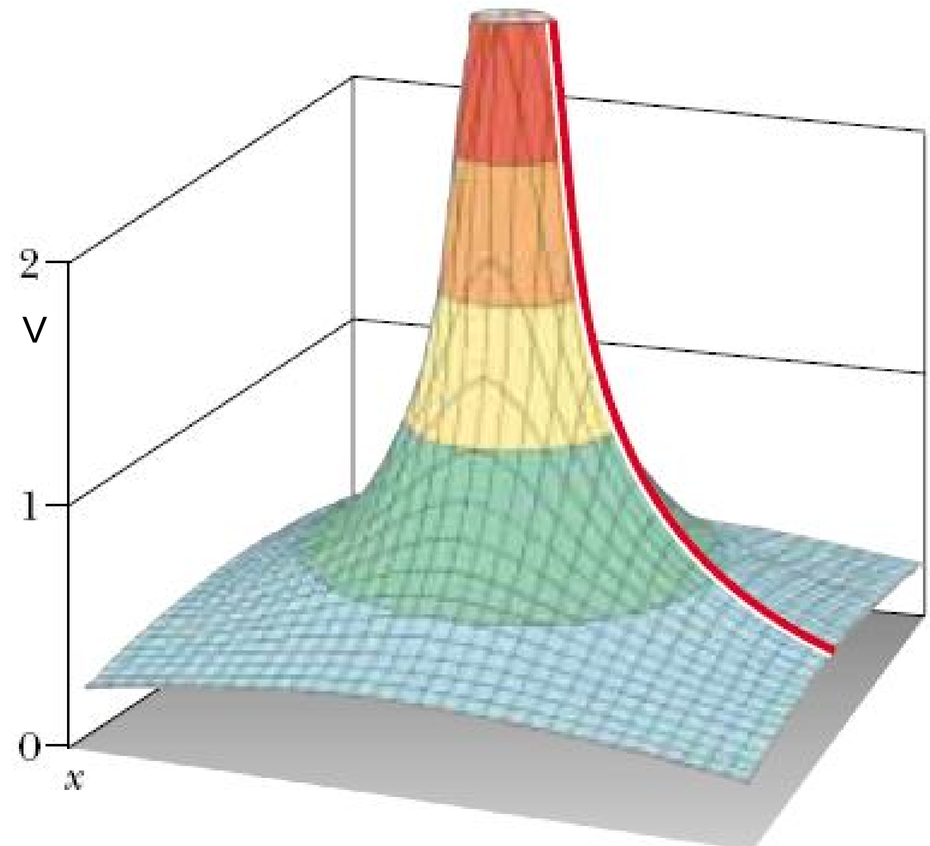
$$\Delta V = (V_b - V_a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Si se considera que $V(r_a = \infty) = 0$

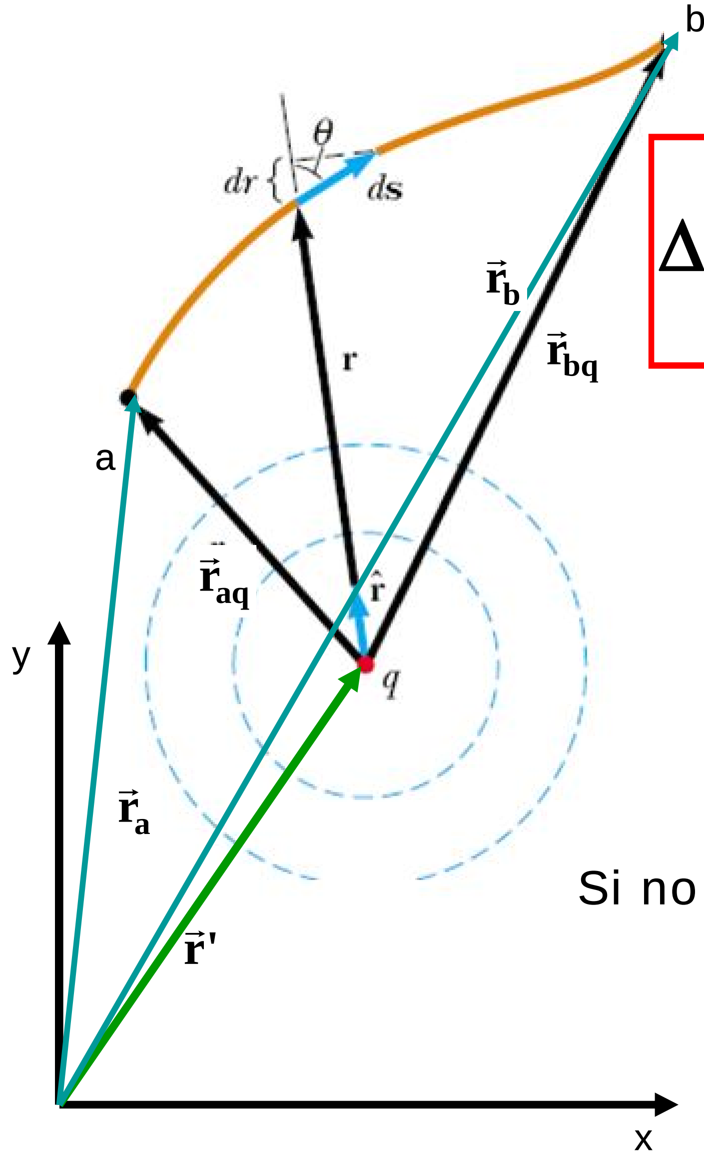
$$\Rightarrow \quad V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$



$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$



Fisica



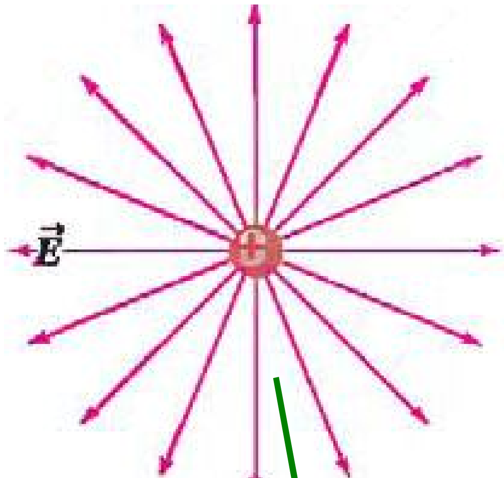
$$\Delta V = (V_b - V_a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{r}_b - \vec{r}'|} - \frac{1}{|\vec{r}_a - \vec{r}'|}\right)$$

$$V(\vec{r}_a) = 0$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{1}{|\vec{r}_b - \vec{r}'|} + cte$$

Si no exite q en el infinito $v(\vec{r}_a = \infty) = 0$

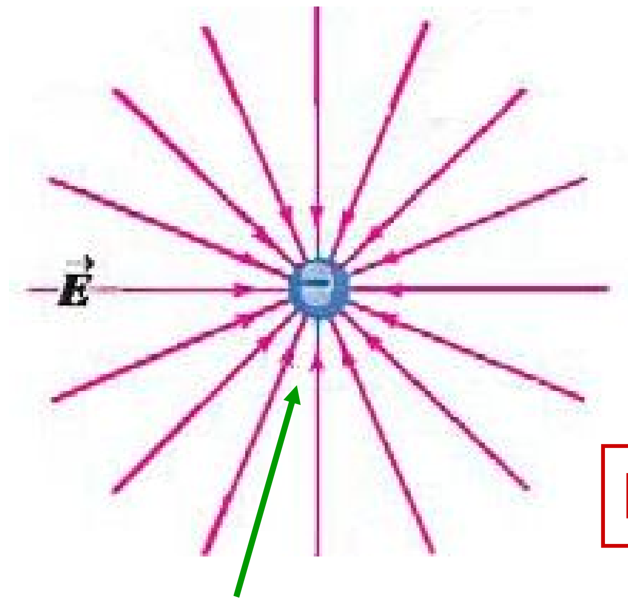
$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{1}{|\vec{r}_b - \vec{r}'|}$$



Si q>0 $$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad \left|\vec{E}(\vec{r})\right| \geq 0 \quad \forall r$$

$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} \qquad V(\vec{r}) \geq 0 \quad \forall r$$

Cuando **r** aumenta **V** disminuye



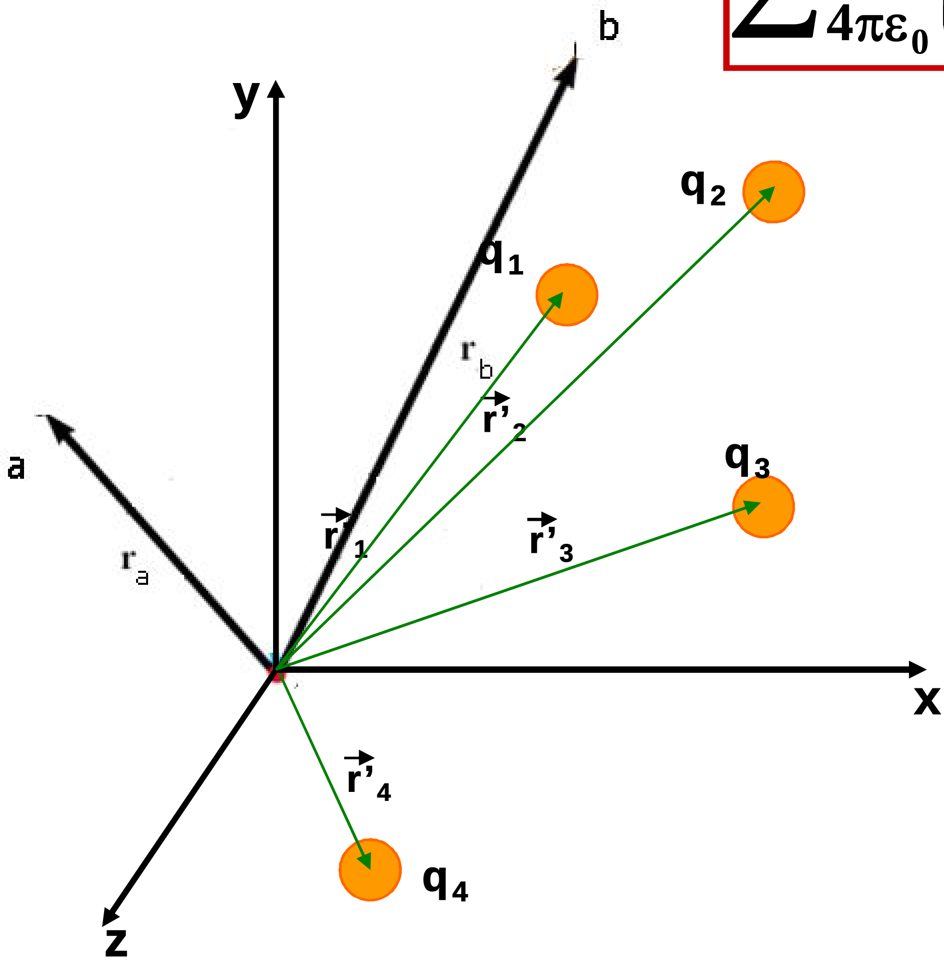
Si q<0 $$\vec{E}(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}\hat{r}$$

$$V(r) = -\frac{|q|}{4\pi\varepsilon_0}\frac{1}{r} \qquad V(\vec{r}) \leq 0 \quad \forall r$$

E apunta hacia donde **V** disminuye

Cuando **r** disminuye **V** disminuye

$$\sum \frac{q_i}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{r}_b - \vec{r}_i\,'|} - \frac{1}{|\vec{r}_a - \vec{r}_i\,'|}\right) = (V_b - V_a) = \Delta V$$



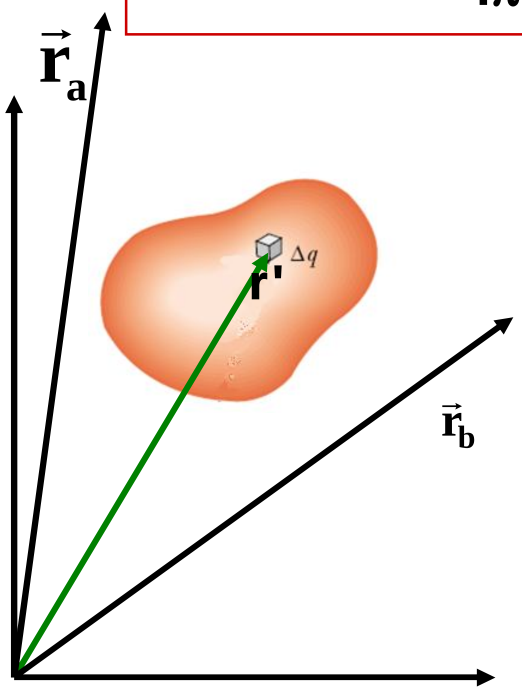
$$r_a \to \infty \qquad V(r_a \to \infty) = O$$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{|\vec{r} - \vec{r}_i\,'|}$$

$$(V_b - V_a) = \Delta V = \int \frac{dq}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{r}_b - \vec{r}'|} - \frac{1}{|\vec{r}_a - \vec{r}'|}\right)$$

$$dq = \rho \, dVol$$

$$V(\vec{r}_b) - V(\vec{r}_a) = \frac{1}{4\pi\varepsilon_0}\left[\iiint \frac{\rho(x',y',z').}{|\vec{r}_b - \vec{r}'|}.dx'dy'dz' - \iiint \frac{\rho(x',y',z').}{|\vec{r}_a - \vec{r}'|}.dx'dy'dz'\right]$$



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\iiint \frac{\rho(x',y',z').}{|\vec{r}_b - \vec{r}'|}.dx'dy'dz'\right] + cte$$

RELACION ENTRE E y V

$$\Delta V = V_b - V_a = -\int_{r_a}^{r_b} \vec{E}.d\vec{l} \quad \Longrightarrow \quad dV = -\vec{E}.d\vec{l}$$

$$\left.\begin{aligned} \vec{E}(\vec{r}) &= E_x(\vec{r})\,\hat{x} + E_y(\vec{r})\,\hat{y} + E_z(\vec{r})\,\hat{z} \\ d\vec{l} &= dx\ \hat{x} + dy\,\hat{y} + dz\ \hat{z} \end{aligned}\right\}$$

$$dV(\vec{r}) = -\left(E_x\,dx + E_y\,dy + E_z\,dz\right)$$

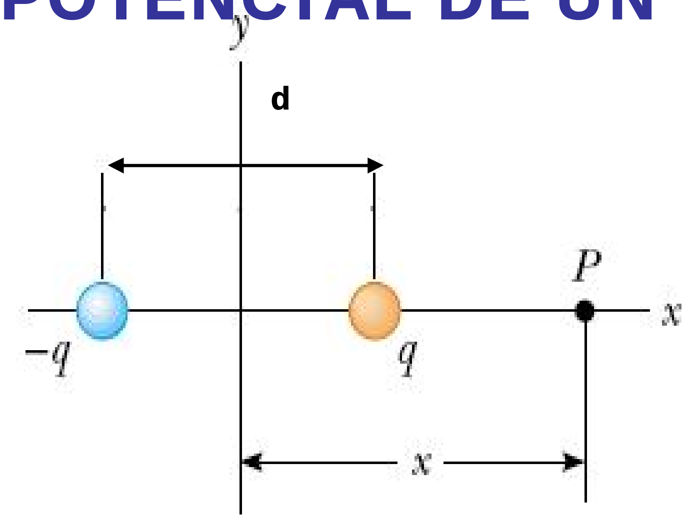
$$dV(\vec{r}) = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$E_x = -\left.\frac{\partial V}{\partial x}\right|_{y,z} \quad , E_y = -\left.\frac{\partial V}{\partial y}\right|_{x,z} \quad , E_z = -\left.\frac{\partial V}{\partial z}\right|_{x,y}$$

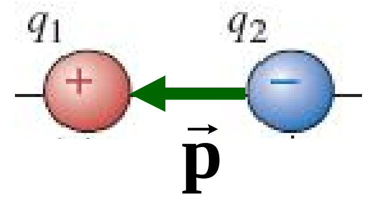
$$\boxed{\vec{E} = -\vec{\nabla} V}$$

$$\boxed{\Delta V = -\int_{r_a}^{r_b} \vec{E}.d\vec{l}}$$

POTENCIAL DE UN DIPOLO

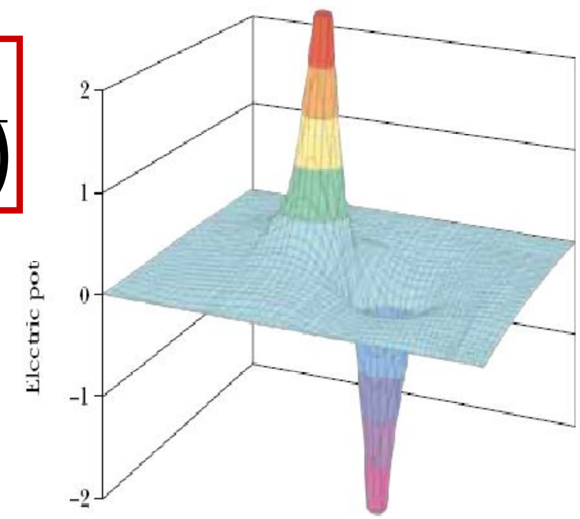


$$\mathbf{p} = \mathbf{q.d}$$



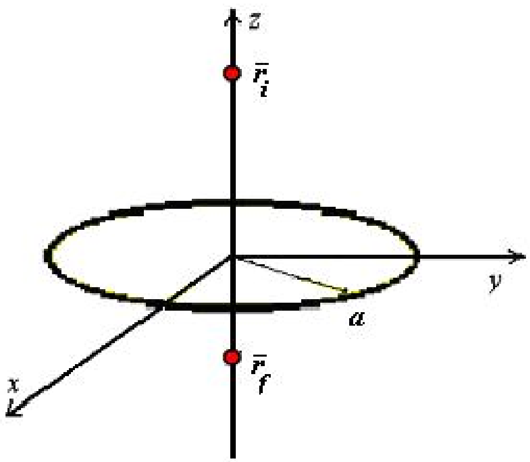
$$\mathbf{V(r)} = \frac{1}{4\pi\varepsilon_0}\sum \frac{\mathbf{q_i}}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_{\mathbf{i}}'|}$$

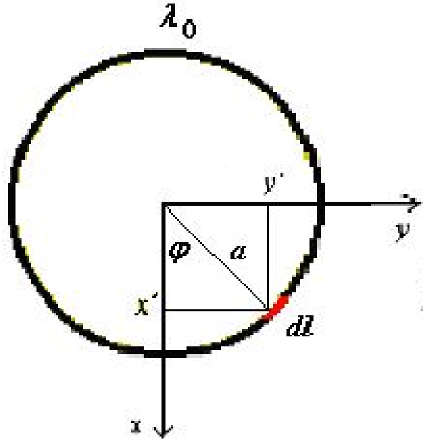
$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x - \mathbf{d}} - \frac{q}{x + \mathbf{d}}\right) = \boxed{\frac{\mathbf{qd}}{4\pi\varepsilon_0}\frac{1}{\left(\mathbf{x}^2 - \mathbf{d}^2\right)}}$$



$$\vec{\mathbf{E}} = -\vec{\nabla}\mathbf{V}$$

Diferencia de potencial entre dos puntos generada por un anillo cargado





$$\mathbf{dq = \lambda dl = \lambda\, a\, d\varphi}$$

$$\vec{r}_i = (0,0,2a)\,;\, \vec{r}_f = (0,0,-a)$$

$$\vec{r}\,' = (x', y', 0) = (a\cos(\varphi), a\sin(\varphi), 0)$$

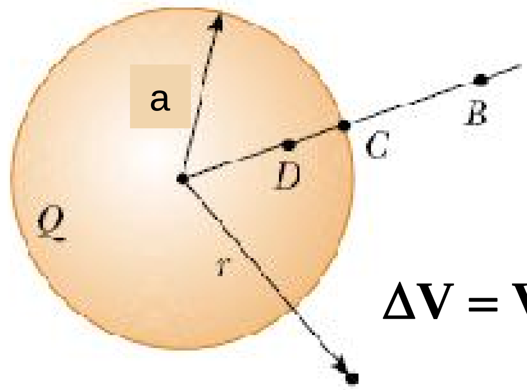
$$(\mathbf{V_f} - \mathbf{V_i}) = \Delta\mathbf{V} = \int \frac{\mathbf{dq}}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{\mathbf{r}}_f - \vec{\mathbf{r}}'|} - \frac{1}{|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}'|}\right)$$

$$|\vec{r}_i - r'| = |(0,0,2a) - (a\cos(\varphi), a\sin(\varphi), 0)| = |(-a\cos(\varphi), -a\sin(\varphi), 2a)| = \sqrt{5}a$$

$$|\vec{r}_f - r'| = |(0,0,-a) - (a\cos(\varphi), a\sin(\varphi), 0)| = |(-a\cos(\varphi), -a\sin(\varphi), -a)| = \sqrt{2}a$$

$$V(\vec{r}_f) - V(\vec{r}_i) = \int_{Anillo} \frac{\lambda_0\, dl}{4\pi\varepsilon_0}\left[\frac{1}{|\vec{r}_f - \vec{r}\,'|} - \frac{1}{|\vec{r}_i - \vec{r}\,'|}\right] = \int_0^{2\pi} \frac{\lambda_0\, a\, d\varphi}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{5}a} - \frac{1}{\sqrt{2}a}\right] = \frac{\lambda_0}{2\varepsilon_0}\left[\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right]$$

POTENCIAL ELECTROSTÁTICO ESFERA UNIFORMEMENTE CARGADA



$$E(r>a)=\frac{\rho\, a^3}{3\varepsilon_0 r^2}=\frac{Q}{4\pi\varepsilon_0 r^2} \qquad E(r<a)=\frac{\rho\, r}{3\varepsilon_0}=\frac{Q\, r}{4\pi\varepsilon_0 R^3}$$

$$\Delta V = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.d\vec{l} \quad = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2} d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$$

$$r_a \to \infty \qquad V(r_a \to \infty) = O \qquad \boxed{V(\vec{r} \geq a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}} \qquad V(c) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}$$

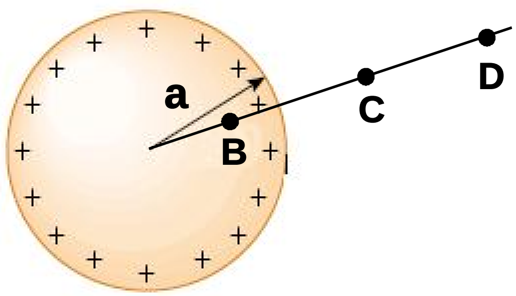
$$\Delta V = V_D - V_C = -\int_{r_C}^{r_D} \vec{E}.d\vec{l} \quad = -\int_{r_C}^{r_D} \frac{Q\, r}{4\pi\varepsilon_0\, a^3} dr = -\frac{Q}{4\pi\varepsilon_0 a^3}\frac{(r^2-R^2)}{2}$$

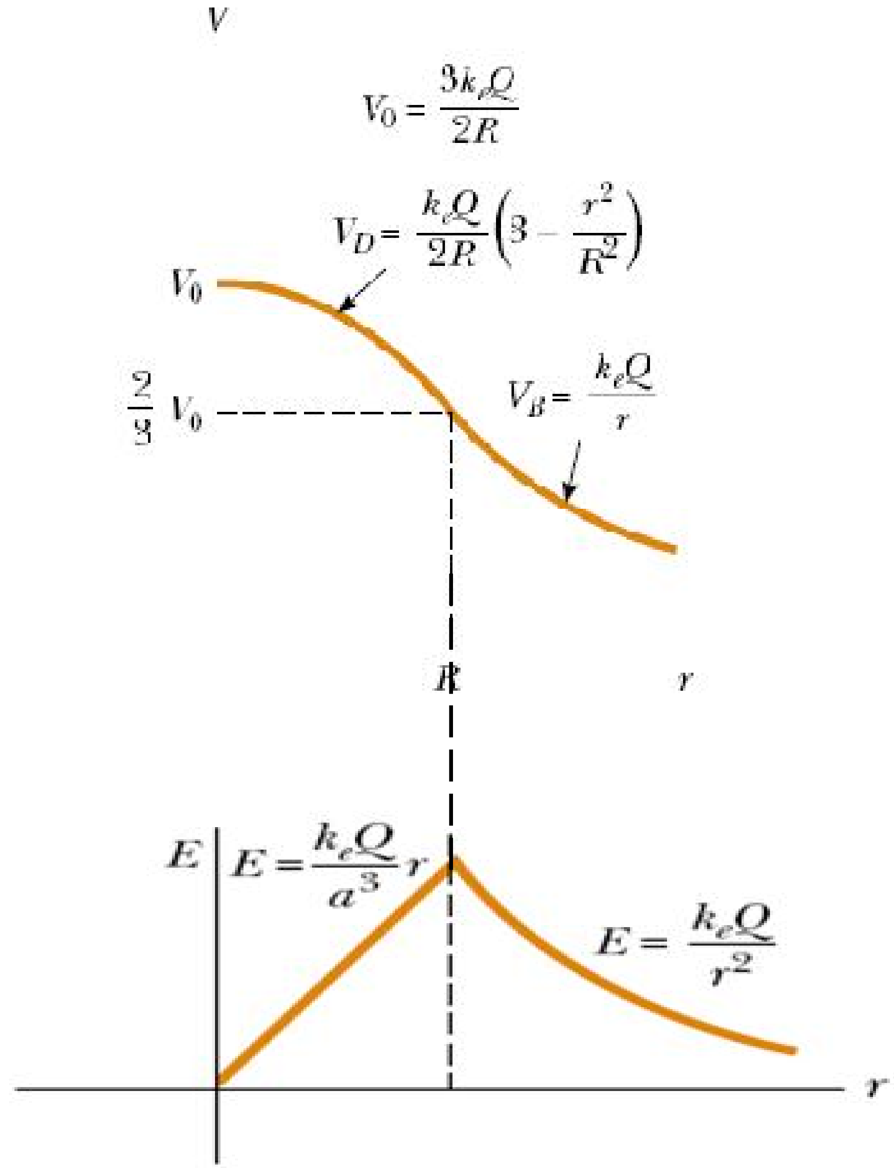
$$V_D = -\frac{Q}{4\pi\varepsilon_0 a^3}\frac{(r^2-a^2)}{2} + V_C = -\frac{Q}{4\pi\varepsilon_0 a^3}\frac{(r^2-a^2)}{2} + \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}$$

$$\boxed{V(r \leq a) = \frac{Q}{8\pi\varepsilon_0}\frac{1}{a^3}(3a^2 - r)}$$

$$\mathbf{V}(\vec{\mathbf{r}} \geq \mathbf{R}) = \frac{\mathbf{Q}}{4\pi\varepsilon_0} \frac{1}{\mathbf{r_B}}$$

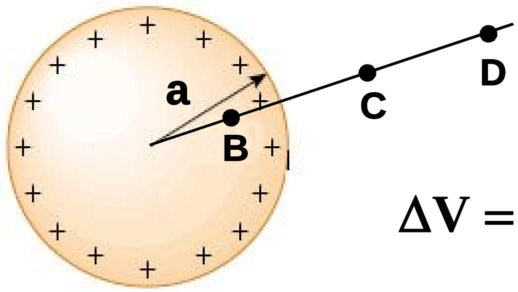
$$\mathbf{V}(\mathbf{r} \leq \mathbf{R}) = \frac{\mathbf{Q}}{8\pi\varepsilon_0} \frac{1}{\mathbf{a}^3} (3\mathbf{a}^2 - \mathbf{r})$$





POTENCIAL ELECTRICO ESFERA UNIFORMEMENTE CARGADA EN SUPERFICIE

$$Q_{total} = \iint \sigma dA = \sigma(4\pi a^2)$$



$$E(r < a) = 0 \qquad E(r > a) = \frac{\sigma a^2}{\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Delta V = V_D - V_C = -\int_{r_D}^{r_c} \vec{E}.d\vec{l} \qquad = -\int_{rD}^{r_C} \frac{Q}{4\pi\varepsilon_0 r^2} d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_C} - \frac{1}{r_D}\right)$$

$$r_a \rightarrow \infty \qquad V(r_D \rightarrow \infty) = O$$

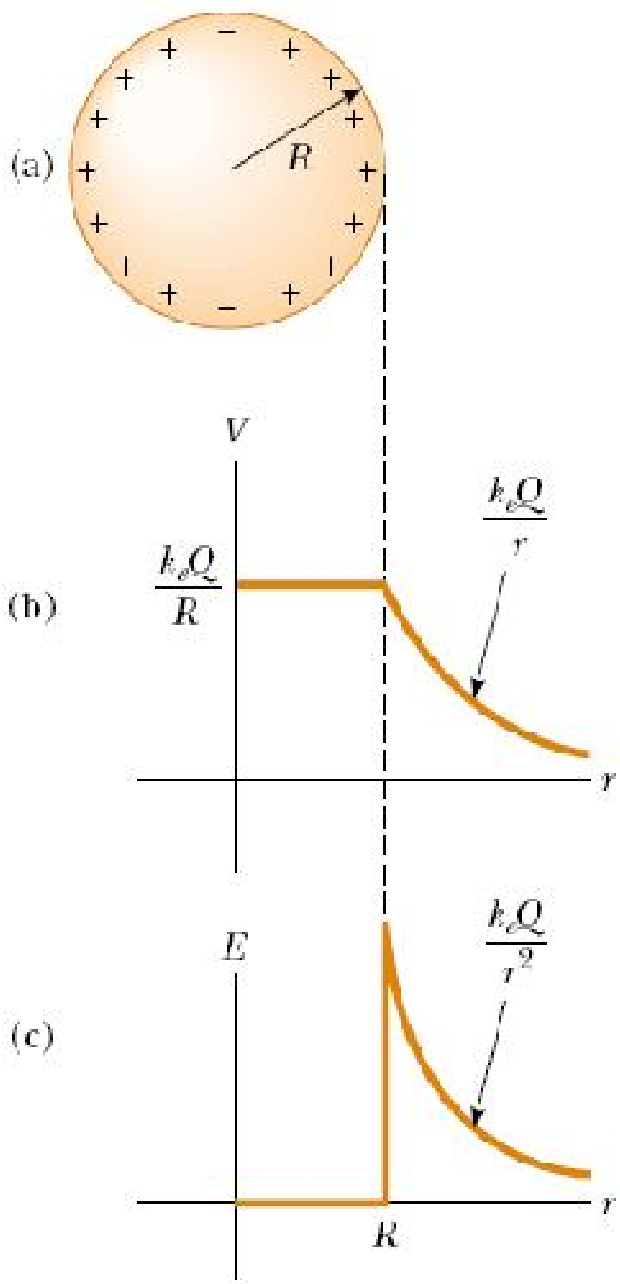
$$\boxed{V(\vec{r} \geq a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_B}}$$

$$V(r = a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}$$

$$\Delta V = V_B - V(r = a) = -\int_{a}^{r_B} \vec{E}.d\vec{l} = 0$$

$$V_B = V(r \leq a) = V_C = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}$$

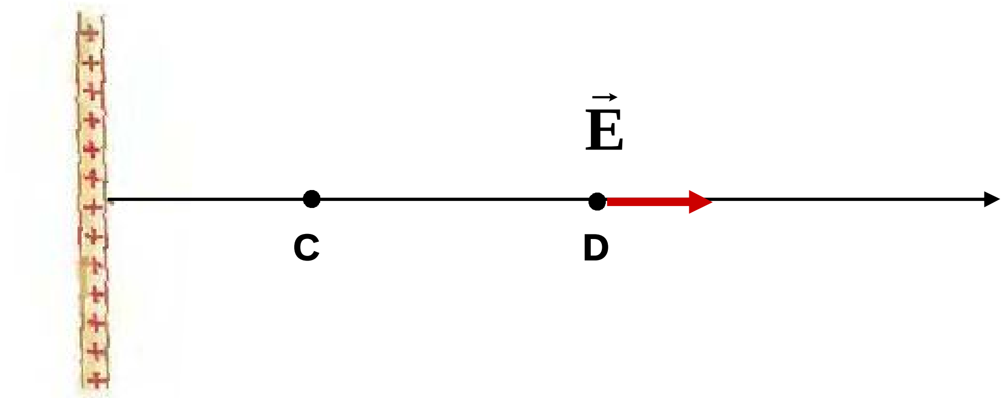
$$\boxed{V(r \leq a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}}$$



$$V(\vec{r} \geq a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_B}$$

$$V(r \leq a) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a}$$

POTENCIAL ELECTRICO HILO INFINITO UNIFORMEMENTE CARGADA



$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\Delta V = V_D - V_C = -\int_{r_D}^{r_C} \vec{E}.d\vec{l} = -\int_{rD}^{r_C} \frac{\lambda}{2\pi\varepsilon_0 r} d\vec{l} = \frac{\lambda}{2\pi\varepsilon_0} Ln\left(\frac{r_D}{r_C}\right)$$

$r_a \to \infty$ $\quad$ $V(r_D \to \infty)\text{diverge}$

Debe definirse el cero de potencial en otro punto

SUPERFICIES EQUIPOTENCIALES

1) Sup. Tridimensionales sobre las cuales **V=cte**

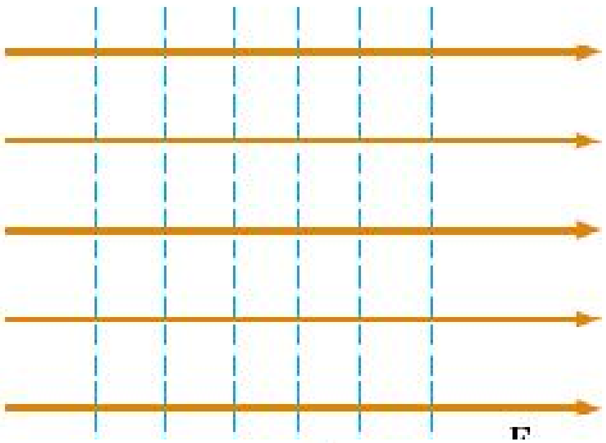
2) Si se desplaza una carga de prueba q_0 desde un punto a otro sobre una equipotencial, como **V=cte**

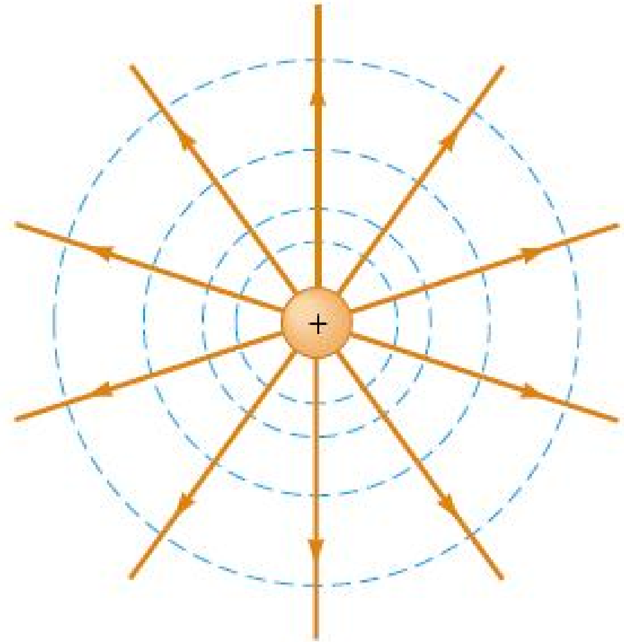
$$U = q_0 V \longrightarrow w_{a\to b} = -\Delta U = 0$$

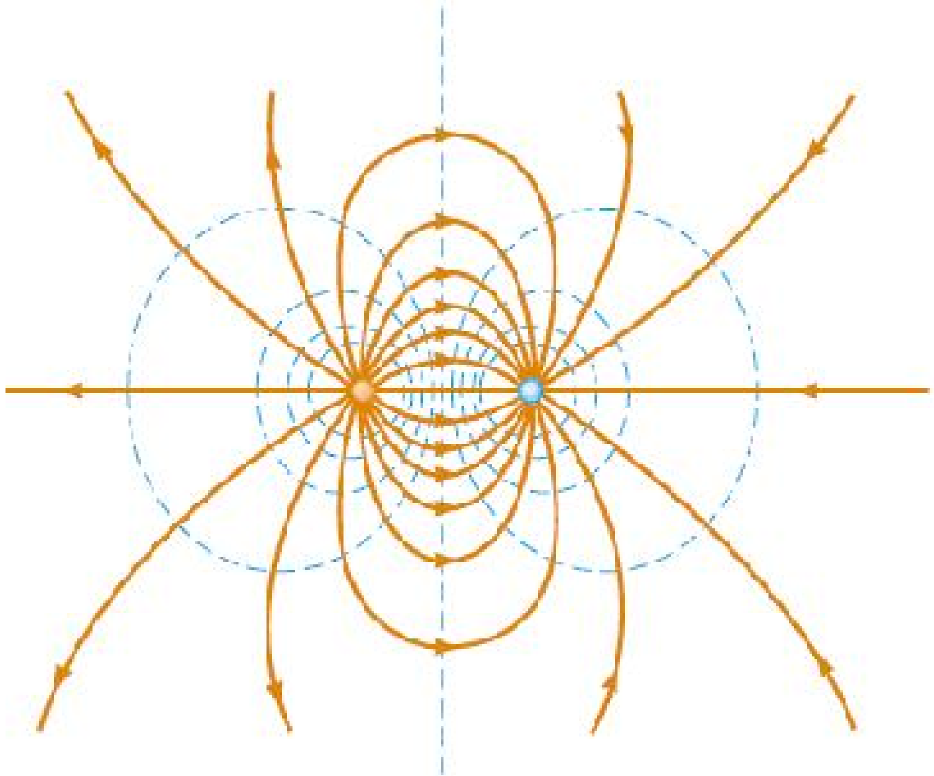
$$w_{a\to b} = q_0 \int_{r_a}^{r_b} \vec{E}.d\vec{l} = 0 \longrightarrow \vec{E}.d\vec{l} = 0 \Rightarrow \vec{E} \text{ perpendicular a } d\vec{l}$$

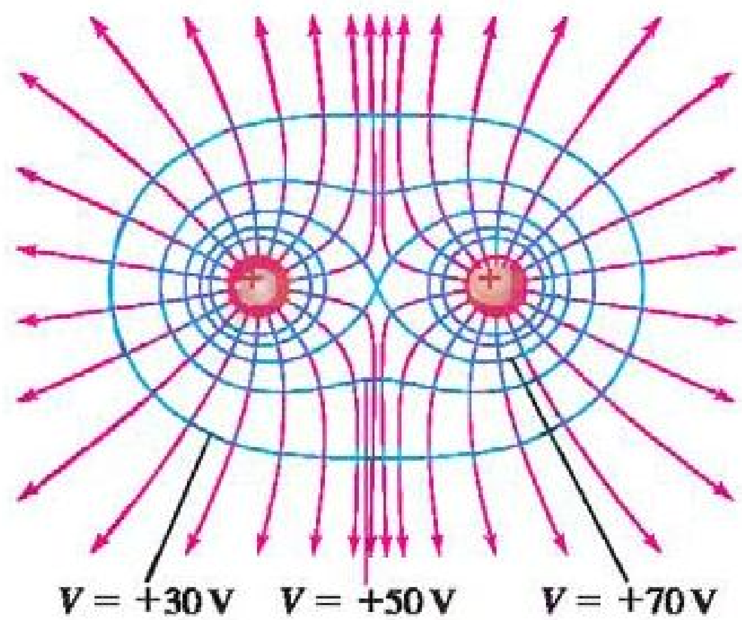
3) Sup. Equipotenciales son perpendicular a **E**

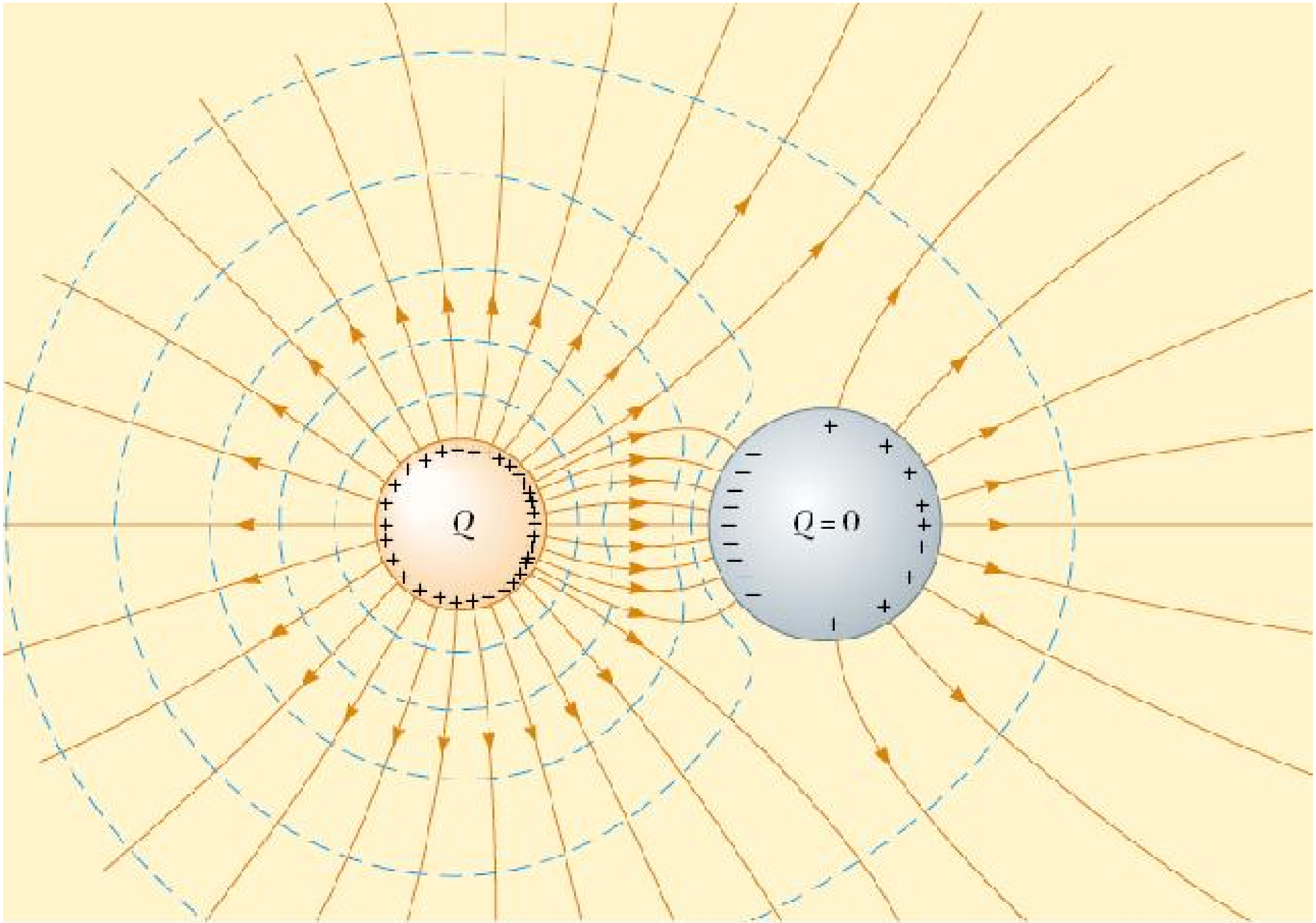
4) Sup. Equipotenciales no se tocan entre si



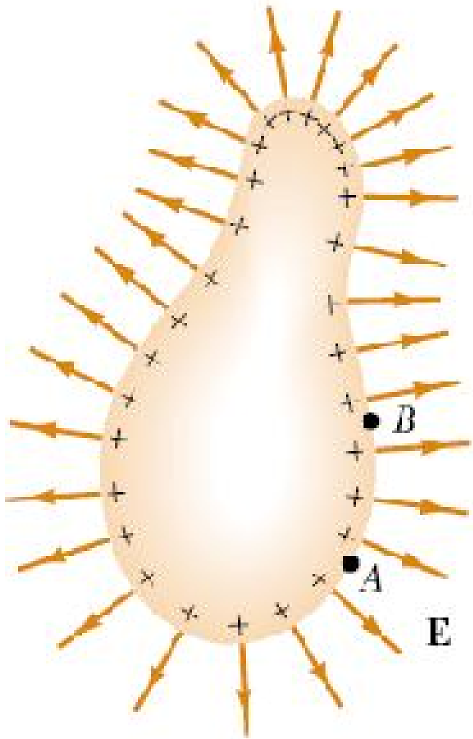


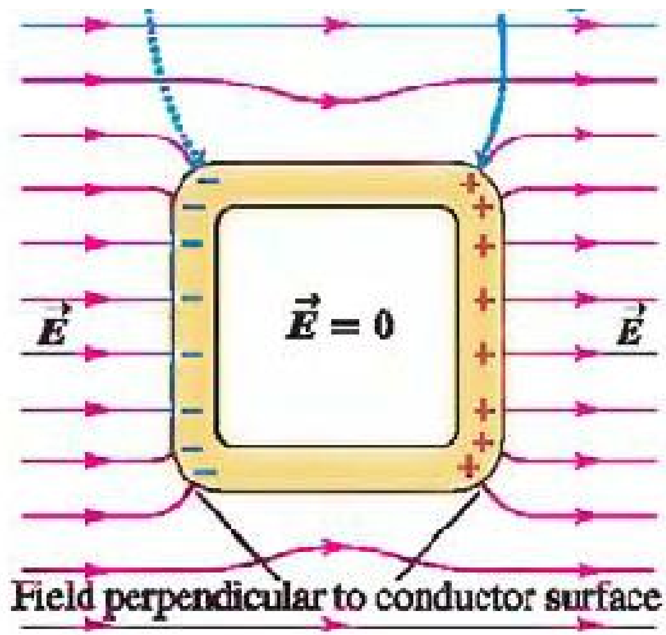


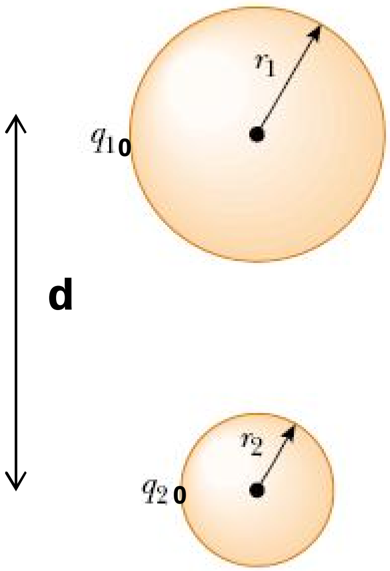

V = +30 V
V = +50 V
V = +70 V


Q
Q = 0

La superficie de un conductor es una equipotencial







$$q_{10} = \iint \sigma_{10}\, dA = 4\pi r_1^2 \sigma_1$$

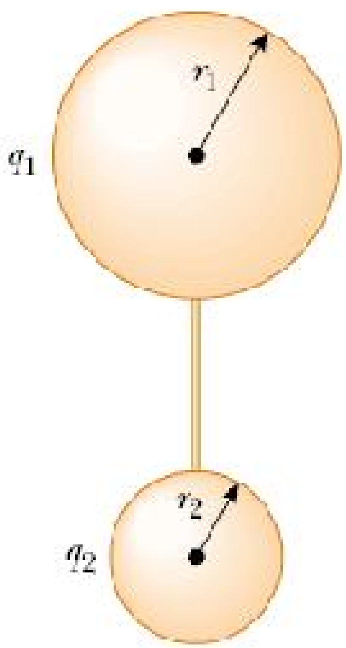
$$d >> r$$

$$q_{20} = \iint \sigma_{20}\, dA = 4\pi r_2^2 \sigma_2$$

$$Q = q_{10} + q_{20}$$

$$V(r_1) = \frac{q_{10}}{4\pi\varepsilon_0}\frac{1}{r_1}$$

$$V(r_2) = \frac{q_{20}}{4\pi\varepsilon_0}\frac{1}{r_2}$$

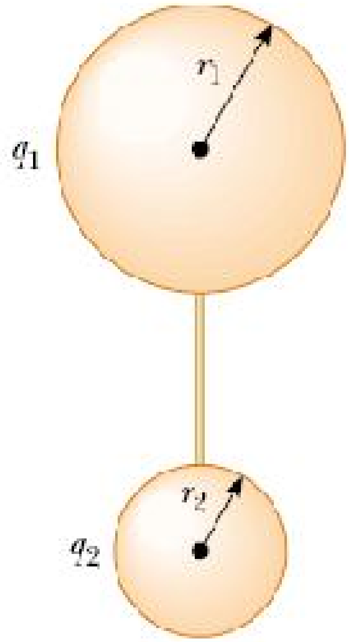


$$Q = q_{10} + q_{20} = q_1 + q_2$$

$$V(r_1) = V(r_2) \longrightarrow \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r_1} = \frac{q_2}{4\pi\varepsilon_0}\frac{1}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \qquad r_1 > r_2 \qquad q_1 > q_2$$

$$q_1 = \frac{Q\, r_1}{r_1 + r_2} \qquad q_2 = \frac{Q\, r_2}{r_1 + r_2}$$



$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \qquad \sigma_1 r_1 = \sigma_2 r_2$$

$$E(r = r_1) = \frac{\sigma_1}{\varepsilon_0} \qquad E(r = r_2) = \frac{\sigma_2}{\varepsilon_0} \qquad E_1 < E_2$$

El campo en un conductor es mayor en las zonas conexas de menor radio de curvatura

Principio de funcionamiento de pararrayos